Recent advances in the theory of nuclear forces

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Abstract. After a brief historical review, we present recent progress in our understanding of nuclear forces in terms of chiral effective field theory.

1. Historical perspective

The theory of nuclear forces has a long history (cf. table 1). Based upon the seminal idea by **Hideki Yukawa** [1], first field-theoretic attempts to derive the nucleon-nucleon (NN) interaction focused on pion-exchange. While the one-pion exchange turned out to be very useful in explaining NN scattering data and the properties of the deuteron [2], multi-pion exchange was beset with serious ambiguities [3, 4]. Thus, the "pion theories" of the 1950s are generally judged as failures—for reasons we understand today: pion dynamics is constrained by chiral symmetry, a crucial point that was unknown in the 1950s.

Historically, the experimental discovery of heavy mesons [5] in the early 1960s saved the situation. The one-boson-exchange (OBE) model [6, 7] emerged which is still the most economical and quantitative phenomenology for describing the nuclear force [8, 9]. The weak point of this model, however, is the scalar-isoscalar "sigma" or "epsilon" boson, for which the empirical evidence remains controversial. Since this boson is associated with the correlated (or resonant) exchange of two pions, a vast theoretical effort that occupied more than a decade was launched to derive the 2π-exchange contribution of the nuclear force, which creates the intermediate range attraction. For this, dispersion theory as well as field theory were invoked producing the Paris [10, 11] and the Bonn [7, 12] potentials.

The nuclear force problem appeared to be solved; however, with the discovery of quantum chromo-dynamics (QCD), all "meson theories" were relegated to models and the attempts to derive the nuclear force started all over again.

The problem with a derivation from QCD is that this theory is non-perturbative in the low-energy regime characteristic of nuclear physics, which makes direct solutions impossible. Therefore, during the first round of new attempts, QCD-inspired quark models [13] became popular. These models were able to reproduce qualitatively some of the gross features of the nuclear force. However, on a critical note, it has been pointed out that these quark-based approaches were nothing but another set of models and, thus, did not represent any fundamental progress. Equally well, one may then stay with the simpler and much more quantitative meson models.

A major breakthrough occurred when the concept of an effective field theory (EFT) was introduced and applied to low-energy QCD. As outlined by Weinberg in a seminal paper [14],

Table 1. Seven Decades of Struggle: The Theory of Nuclear Forces

1935	**Yukawa: Meson Theory**
1950's	*The "Pion Theories"* One-Pion Exchange: o.k. Multi-Pion Exchange: disaster
1960's	Many pions $\equiv$ multi-pion resonances: $\boldsymbol{\sigma}$, $\boldsymbol{\rho}$, $\boldsymbol{\omega}$, ... The One-Boson-Exchange Model
1970's	Refine meson theory: Sophisticated $\mathbf{2\pi}$ exchange models (Stony Brook, Paris, Bonn)
1980's	Nuclear physicists discover **QCD** Quark Cluster Models
1990's and beyond	Nuclear physicists discover **EFT** Weinberg, van Kolck **Back to Meson Theory!** *But, with Chiral Symmetry*

one has to write down the most general Lagrangian consistent with the assumed symmetry principles, particularly the (broken) chiral symmetry of QCD. At low energy, the effective degrees of freedom are pions and nucleons rather than quarks and gluons; heavy mesons and nucleon resonances are "integrated out". So, the circle of history is closing and we are *back to Yukawa's meson theory*, except that we have learned to add one important refinement to the theory: broken chiral symmetry is a crucial constraint that generates and controls the dynamics and establishes a clear connection with the underlying theory, QCD.

It is the purpose of the remainder of this paper to describe the EFT approach to nuclear forces in more detail.

2. Chiral perturbation theory and the hierarchy of nuclear forces

The chiral effective Lagrangian is given by an infinite series of terms with increasing number of derivatives and/or nucleon fields, with the dependence of each term on the pion field prescribed by the rules of broken chiral symmetry. Applying this Lagrangian to NN scattering generates an unlimited number of Feynman diagrams. However, Weinberg showed [15] that a systematic expansion exists in terms of $(Q/\Lambda_\chi)^\nu$, where Q denotes a momentum or pion mass, $\Lambda_\chi \approx 1$ GeV is the chiral symmetry breaking scale, and $\nu \geq 0$ (cf. figure 1). This has become known as chiral perturbation theory (χPT). For a given order ν, the number of terms is finite and calculable; these terms are uniquely defined and the prediction at each order is model-independent. By going to higher orders, the amplitude can be calculated to any desired accuracy.

Following the first initiative by Weinberg [15], pioneering work was performed by Ordóñez, Ray, and van Kolck [16, 17] who constructed a NN potential in coordinate space based upon χPT at next-to-next-to-leading order (NNLO; $\nu = 3$). The results were encouraging and many researchers became attracted to the new field. Kaiser, Brockmann, and Weise [18] presented the first model-independent prediction for the NN amplitudes of peripheral partial waves at NNLO.

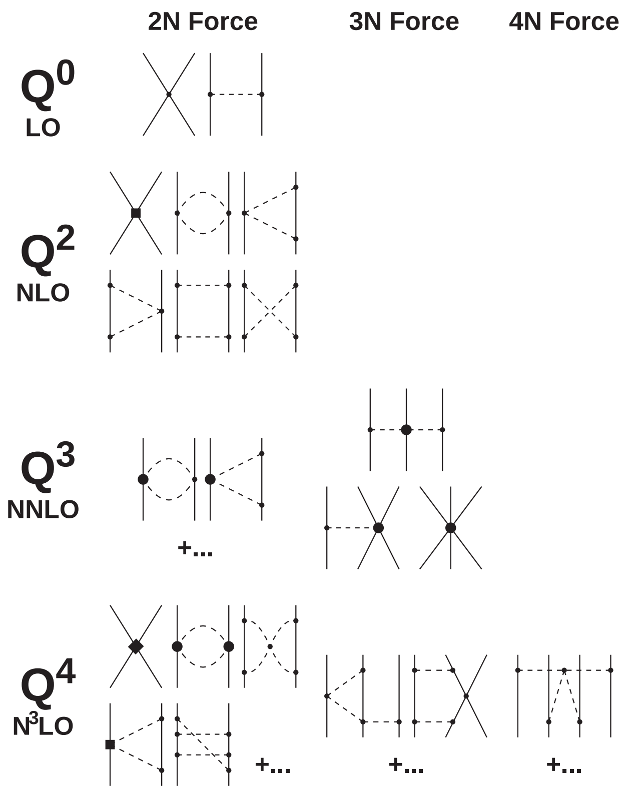

Figure 1. Hierarchy of nuclear forces in χPT. Solid lines represent nucleons and dashed lines pions. Further explanations are given in the text.

Epelbaum *et al.* [19] developed the first momentum-space NN potential at NNLO, and Entem and Machleidt [20] presented the first potential at N^{3}LO ($\nu = 4$).

In χPT, the NN amplitude is uniquely determined by two classes of contributions: contact terms and pion-exchange diagrams. There are two contacts of order Q^0 [$\mathcal{O}(Q^0)$] represented by the four-nucleon graph with a small-dot vertex shown in the first row of figure 1. The corresponding graph in the second row, four nucleon legs and a solid square, represent the seven contact terms of $\mathcal{O}(Q^2)$. Finally, at $\mathcal{O}(Q^4)$, we have 15 contact contributions represented by a four-nucleon graph with a solid diamond.

Now, turning to the pion contributions: At leading order [LO, $\mathcal{O}(Q^0)$, $\nu = 0$], there is only the wellknown static one-pion exchange, second diagram in the first row of figure 1. Two-pion exchange (TPE) starts at next-to-leading order (NLO, $\nu = 2$) and all diagrams of this leading-order two-pion exchange are shown. Further TPE contributions occur in any higher order. Of this sub-leading TPE, we show only two representative diagrams at NNLO and three diagrams at N^{3}LO. While TPE at NNLO was known for a while [16, 18, 19], TPE at N^{3}LO has been calculated only recently by Kaiser [21]. All 2π exchange diagrams/contributions up to N^{3}LO are summarized in a pedagogical and systematic fashion in Ref. [22] where the model-independent results for NN scattering in peripheral partial waves are also shown.

Finally, there is also three-pion exchange, which shows up for the first time at N^{3}LO (two loops; one representative 3π diagram is included in figure 1). In Ref. [23], it was demonstrated that the 3π contribution at this order is negligible.

One important advantage of χPT is that it makes specific predictions also for many-body forces. For a given order of χPT, 2NF, 3NF, . . . are generated on the same footing (cf. figure 1).

Table 2. χ^2/datum for the reproduction of the 1999 *np* database below 290 MeV by various *np* potentials. ($\Lambda = 500$ MeV in all chiral potentials.)

Bin (MeV)	# of data	N^3LO^a	$NNLO^b$	NLO^b	$AV18^c$
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

[a]Reference [20]. [b]Reference [26]. [c]Reference [27].

At LO, there are no 3NF, and at next-to-leading order (NLO), all 3NF terms cancel [15, 24]. However, at NNLO and higher orders, well-defined, nonvanishing 3NF occur [24, 25]. Since 3NF show up for the first time at NNLO, they are weak. Four-nucleon forces (4NF) occur first at N^3LO and, therefore, they are even weaker.

3. Chiral NN potentials

The two-nucleon system is non-perturbative as evidenced by the presence of shallow bound states and large scattering lengths. Weinberg [15] showed that the strong enhancement of the scattering amplitude arises from purely nucleonic intermediate states. He therefore suggested to use perturbation theory to calculate the NN potential and to apply this potential in a scattering equation (Lippmann-Schwinger or Schrödinger equation) to obtain the NN amplitude. We follow this philosophy.

Chiral perturbation theory is a low-momentum expansion. It is valid only for momenta $Q \ll \Lambda_\chi \approx 1$ GeV. Therefore, when a potential is constructed, all expressions (contacts and irreducible pion exchanges) are multiplied with a regulator function,

$$\exp\left[-\left(\frac{p}{\Lambda}\right)^{2n} - \left(\frac{p'}{\Lambda}\right)^{2n}\right] , \tag{1}$$

where p and p' denote, respectively, the magnitudes of the initial and final nucleon momenta in the center-of-mass frame; and $\Lambda \ll \Lambda_\chi$. The exponent $2n$ is to be chosen such that the regulator generates powers which are beyond the order at which the calculation is conducted.

The χ^2/datum for the fit of the *np* data below 290 MeV is shown in table 2, and the corresponding one for *pp* is given in table 3. The χ^2 tables show the quantitative improvement

Table 3. χ^2/datum for the reproduction of the 1999 *pp* database below 290 MeV by various *pp* potentials. ($\Lambda = 500$ MeV in all chiral potentials.)

Bin (MeV)	# of data	N^3LO^a	$NNLO^b$	NLO^b	$AV18^c$
0–100	795	1.05	6.66	57.8	0.96
100–190	411	1.50	28.3	62.0	1.31
190–290	851	1.93	66.8	111.6	1.82
0–290	2057	1.50	35.4	80.1	1.38

[a]Reference [20]. [b]See footnote [28]. [c]Reference [27].

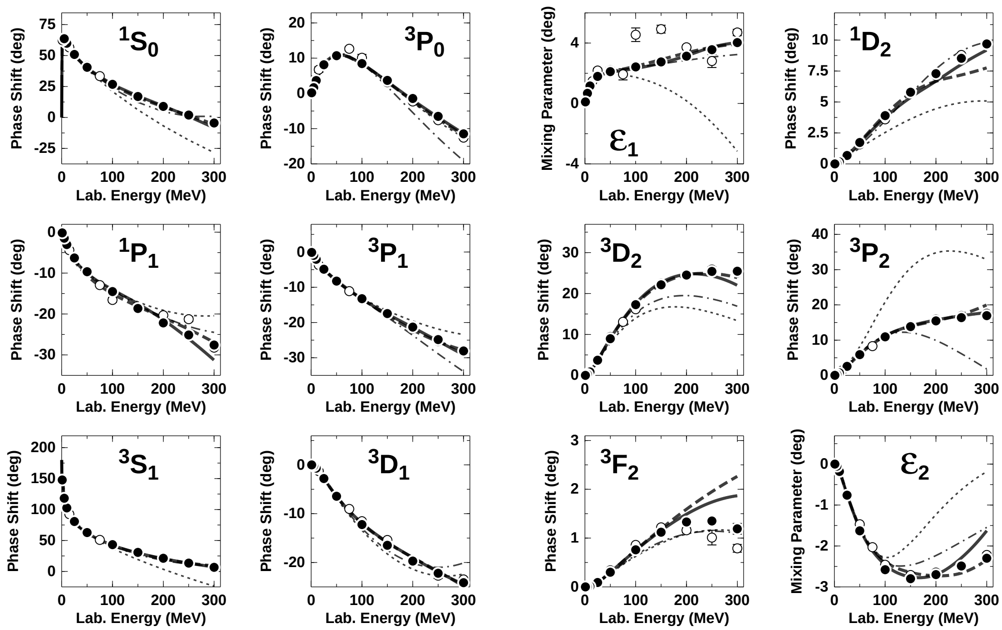

Figure 2. *np* phase parameters below 300 MeV lab. energy for partial waves with $J \leq 2$. The thick solid (dashed) line is the result by Entem and Machleidt [20] at N^{3}LO using $\Lambda = 500$ MeV ($\Lambda = 600$ MeV). The thin dotted and dash-dotted lines are the phase shifts at NLO and NNLO, respectively, as obtained by Epelbaum *et al.* [26] using $\Lambda = 500$ MeV. The solid dots show the Nijmegen multienergy *np* phase shift analysis [29], and the open circles are the GWU/VPI single-energy *np* analysis SM99 [30].

of the NN interaction order by order in a dramatic way. Even though there is considerable improvement when going from NLO to NNLO, it is clearly seen that N^{3}LO is needed to achieve an accuracy comparable to the phenomenological high-precision Argonne V_{18} potential [27]. Note that proton-proton data have, in general, smaller errors than *np* data which explains why the *pp* χ^2 are always larger.

The phase shifts for *np* scattering below 300 MeV lab. energy are displayed in figure 2. What the χ^2 tables revealed, can be seen graphically in this figure. The $^3P_2$ phase shifts are a particularly good example: NLO (dotted line) is clearly poor. NNLO (dash-dotted line) brings improvement and describes the data up to about 100 MeV. The difference between the NLO and NNLO curves is representative for the uncertainty at NLO and, similarly, the difference between NNLO and N^{3}LO reflects the uncertainty at NNLO. Obviously, at N^{3}LO ($\Lambda = 500$ MeV, thick solid line) we have a good description up to 300 MeV. An idea of the uncertainty at N^{3}LO can be obtained by varying the cutoff parameter Λ. The thick dashed line is N^{3}LO using $\Lambda = 600$ MeV. In most cases, the latter two curves are not distinguishable on the scale of the figures. Noticeable differences occur only in $^1D_2$, $^3F_2$, and ϵ_2 above 200 MeV.

4. Conclusions

The EFT approach to nuclear forces is a modern refinement of Yukawa's meson theory. It represents a scheme that has an intimate relationship with QCD and allows to calculate nuclear forces to any desired accuracy. Moreover, nuclear two- and many-body forces are generated on the same footing.

At N^{3}LO [20], the accuracy is achieved that is necessary and sufficient for reliable nuclear

structure calculations. First calculations applying the N^3LO potential have produced promising results [31, 32, 33, 34, 35, 36].

The 3NF at NNLO is known [25] and has had first successful applications [31]. The 3NF and 4NF at N^3LO is presently under construction.

In summary, the stage is set for many years of exciting nuclear structure research that is more consistent than anything we had before.

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